PACE Electrical Engineering

Create a Formula From a Truth Table

Introduction

- There may be times when you want to create a formula for a digital circuit (and then the digital circuit itself) from a truth table
  - The easiest way to characterize the function you want may be by creating the truth table
- There are various techniques for doing this. We will present one in this presentation

Example

- Suppose you have a set of sensors that can each detect whether the up face on a die is an even or odd number

- You also have a set of three dice
- Your goal is to create a circuit that will determine whether the sum of the up faces of the three dice is even or odd
- Let A, B, and C be inputs from the sensors into the circuit
  - Each corresponds to one die
  - The value of each input is 1 if the corresponding die is odd and 0 if it is even
- Let D be the output of the circuit. D is 1 if the sum of the dice is odd

Example (cont.)

- We will create a truth table for D
- Note that the sum of three dice is odd if exactly one of them is odd or if all three are odd
  - Make up some examples to convince yourself that this statement is true

Truth Table for D

A	B	C	D
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Technique

- We concentrate on the rows in the truth table for which the output is 1 (D=1).

A	B	C	D
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Technique (cont.)

- D will be expressed as a sum of terms, one for each of the rows for which D is 1
- From the truth table we know that D equals 1 when
  - A equals 0 and B equals 0 and C equals 1, or
  - A equals 0 and B equals 1 and C equals 0, or
  - A equals 1 and B equals 0 and C equals 0, or
  - A equals 1 and B equals 1 and C equals 1
- This is equivalent to saying D equals 1 when
  - $\overline{A}$ equals 1 and $\overline{B}$ equals 1 and C equals 1, or
  - $\overline{A}$ equals 1 and B equals 1 and $\overline{C}$ equals 1, or
  - A equals 1 and $\overline{B}$ equals 1 and $\overline{C}$ equals 1, or
  - A equals 1 and B equals 1 and C equals 1

Answer

- Using the last description, we can create a formula which is a sum of four terms. Each term corresponds to a row in the truth table for which D = 1 and represents the combination of values of A, B, and C which result in D = 1. The formula is

$$D = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + ABC$$

A Simpler Example

A	B	C
0	0	0
0	1	1
1	0	0
1	1	1

$$C = \overline{A}B + AB$$